

Single View Metrology Along Orthogonal Directions

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Abstract—In this paper, we describe how 3D metric measurements can be determined from a single uncalibrated image, when only minimal geometric information are available in the image. The minimal information just is orthogonal vanishing points. Given such limited information, we show that the length ratios on different orthogonal directions can be directly computed. The exciting discovery of the method seems to oppose common senses: Usually, in the calibration process, all edge-lengths of cuboid are known, in this paper, cuboid edge-lengths are unknown but its edge-lengths ratios can be recovered from image. 3D metric measurements can be directly computed from the image using our linear method.

Keywords-3D metric measurements; Orthogonal vanishing points; Single uncalibrated image; Linear method;

I. INTRODUCTION

In recent years, metrology from uncalibrated images is becoming increasing interest for a variety of approaches, such as 3D graphical modelling, measurements in forensic images, and image-based rendering. In this paper, We describe how to compute the metric 3D geometry of a scene from a single image. Because the image of real world captured by a camera consists of perspective distortion, rectangular features (such as windows, doors, walls) will not appear as rectangles. Therefore, we have to perform perspective correction in images to compute the Euclidean properties like length, angle and parallelism. Moreover, as pointed in [10], rectification is crucial to reveal correct structure of the captured scene. However, existing methods for computing the exact aspect ratios of rectangles from uncalibrated images are difficult, and these methods can not be directly extended to 3D case for recovering the ratios of cuboid's length, width and height in single images. Our method just needs orthogonal vanishing points to recover the aspect ratio of rectangle or ratios of cuboid's length, width and height, then we can obtain the metric measurements from uncalibrated images.

Related Work: 1) Camera calibration from vanishing points: Camera calibration is a fundamental task in computer vision. Caprile and Torre [2] reported how to get robust camera calibration from three orthogonal vanishing points in images, after that, many efficient methods [1], [3], [6],

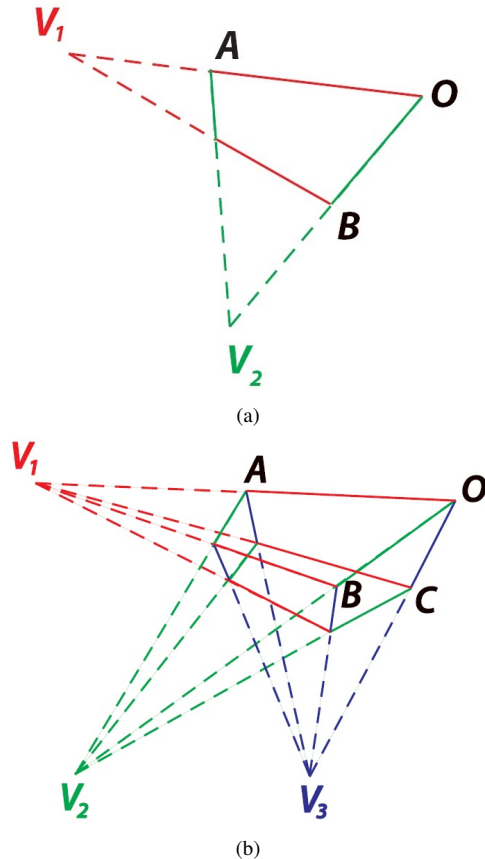


Figure 1. (a) Given two orthogonal vanishing points V_1, V_2 , we can compute $OA:OB$. (b) Given orthogonal vanishing points V_1, V_2 and V_3 , we can compute $OA:OB:OC$.

[14], [15] have been proposed to calibrate the camera from vanishing points.

2) Metrology from images: Single view metrology has been addressed numerous times in both photogrammetry and computer vision [8], [9], [11], [12]. Criminisi et al [4] extended previous results and proposed a classic method for single view metrology, showing that affine scene structure can be recovered from a single uncalibrated image. Their approach recovers 3D affine measurements using a vanishing line of a reference plane, and a vanishing point for a

direction not parallel to the plane. However, the limitation is that they require three reference distances in mutually orthogonal directions to be available on the image to recover metric measurements. Foroosh et al [5] compute affine measurements using two views, and the length ratio of two objects perpendicular to the reference plane can be expressed. They also only compute the measurements in one direction. We can recover the length ratios of line segments that lie on different orthogonal directions. Thus only one reference distance is needed to recover the 3D metric measurements from a single uncalibrated image, which is an obvious advantage compared with other methods.

In the remaining of this paper, section 2 will introduce the camera calibration using vanishing points. In section 3, we describe how to compute the metric measurements; then the evaluations and experiments are presented in section 4 and 5; the conclusion is in the section 6.

II. CAMERA CALIBRATION

Under perspective projection, a 3D point X is projected to an image point $x = PX$ (equal up to scale), where P is the camera projection matrix given as

$$P = K[R | t] = K[r_1 \ r_2 \ r_3 | t] \quad (1)$$

r_1, r_2, r_3 are the columns of the 3×3 rotation matrix R , t is the translation vector, and K is the intrinsic matrix of camera in form of

$$K = \begin{pmatrix} f_x & s & u_0 \\ 0 & \alpha f_x & v_0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

where f_x is the focal length corresponding to the x axe of camera coordinate; α is the aspect ratio and s refers to the skew factor; $[u_0 \ v_0]^T$ is principal point of camera, see more details in [7]. K can be simplified to only three parameters assuming unit aspect ratio and zero skew,

$$K = \begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3)$$

f is the focal length and $[u_0 \ v_0]^T$ is principal point of camera, more details in [7].

Denote three orthogonal directions in 3D space as $e_1 = [1 \ 0 \ 0]^T$, $e_2 = [0 \ 1 \ 0]^T$, $e_3 = [0 \ 0 \ 1]^T$, corresponding vanishing points on the image are $v_1 = KRe_1$, $v_2 = KRe_2$, $v_3 = KRe_3$.

We have

$$e_1^T e_2 = v_1^T K^{-T} K^{-1} v_2 = 0 \quad (4)$$

Similarly $v_1^T K^{-T} K^{-1} v_3 = 0$ and $v_2^T K^{-T} K^{-1} v_3 = 0$. Only three parameters of K are unknown, three detected vanishing points provide three independent constraints on K , so K can be obtained directly.

Each vanishing point is proportional to the column of the rotation matrix, $q_1 = K^{-1}v_1, q_2 = K^{-1}v_2, q_3 = K^{-1}v_3$. Matrix $Q = [q_1 \ q_2 \ q_3]$ is obtained, but Q generally does not satisfy the properties of a rotation matrix. We use the algorithm in [16] to approximate the best rotation matrix $R = [r_1 \ r_2 \ r_3]$ from Q . That is, we solve the following problem:

$$\min_R \| R - Q \|^2 \quad (R^T R = I) \quad (5)$$

Let the singular value decomposition (SVD) of Q be USV^T , then the solution is $R = UV^T$.

However, there may be only two vanishing points on image. In this case, points are all in plane $z = 0$, the camera projection matrix reduces to the 3×3 homography given by $P = K[r_1 \ r_2 | t]$. Assume principal point lies on the center of image, from $(v_1(x) - u_0)(v_2(x) - u_0) + (v_1(y) - v_0)(v_2(y) - v_0) + f^2 = 0$. f also can be computed. Compute $q_1 = K^{-1}v_1, q_2 = K^{-1}v_2$, from matrix $Q = [q_1 \ q_2]$, $R = [r_1 \ r_2]$ is obtained.

For the translation vector t . If the original point is projected to the image point O , then $O = K[R | t][0 \ 0 \ 0 \ 1] = Kt$. Therefore, $t = K^{-1}O$.

III. MEASUREMENTS BETWEEN ORTHOGONAL DIRECTIONS

A rectangle of real world is projected on image like Figure 1(a), parallel edges intersect at vanishing points, we want to recover its ratio of length and width. The ratio is given by

$$\frac{OA}{OB} = \frac{(A, O)(E_1, V_1)(E_2, O)(B, V_2)}{(A, V_1)(E_1, O)(E_2, V_2)(B, O)} \quad (6)$$

where (\cdot) denotes the Euclidean distance between two points, $E_1 = K[R|t][1 \ 0 \ 0 \ 1]^T$ and $E_2 = K[R|t][0 \ 1 \ 0 \ 1]^T$. The equality follows from the invariance of the cross-ratio under projective transformation. Because we have obtained K, R and t in Section 2, the Eq (5) can be directly solved.

In practice, we can obtain the solution more efficiently. When compute K and R from vanishing points, $\frac{OA}{OB}$ and t can be obtained at once. O, A and B are projected by the camera, given by

$$[O \ A \ B] = K[R|t] \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad (7)$$

O, A, B are chosen image points. Let

$$X = \begin{pmatrix} -1 & 0 & x_O & 0 & 0 \\ 0 & -1 & y_O & 0 & 0 \\ -1 & 0 & x_A & x_A r_{31} - r_{11} & 0 \\ -1 & 0 & y_A & y_A r_{31} - r_{21} & 0 \\ -1 & 0 & x_B & 0 & x_B r_{32} - r_{12} \\ -1 & 0 & y_B & 0 & y_B r_{32} - r_{22} \end{pmatrix}$$

and $\delta = [t_1 \ t_2 \ t_3 \ a \ b]^T$, we have $X\delta = 0$. Since $\text{rank}(X) = 5$, δ can be computed from the equation. The

eigenvector associated with the smallest eigenvalue of $X^T X$ is δ . a , b and $t = [t_1 \ t_2 \ t_3]^T$ are linearly obtained (up to scale). The ratio $\frac{a}{b}$ is the aspect ratio of rectangle.

Moreover, we can define projective coordinate system on the reference plane. Compute the cross-ratios of image points with vanishing points V_1, V_2 and unit point E_1, E_2 , we can obtain their 3D positions on the 3D plane. See more details in [13]. Thus, the measurements on the reference plane can be completely computed.

In 3D case, like Figure 1(b), we can similarly obtain

$$[O \ A \ B \ C] = K[R|t] \begin{pmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (8)$$

Let $X =$

$$\begin{pmatrix} -1 & 0 & x_O & 0 & 0 & 0 \\ 0 & -1 & y_O & 0 & 0 & 0 \\ -1 & 0 & x_A & x_A r_{31} - r_{11} & 0 & 0 \\ -1 & 0 & y_A & y_A r_{31} - r_{21} & 0 & 0 \\ -1 & 0 & x_B & 0 & x_B r_{32} - r_{12} & 0 \\ -1 & 0 & y_B & 0 & y_B r_{32} - r_{22} & 0 \\ -1 & 0 & x_C & 0 & 0 & x_C r_{33} - r_{13} \\ -1 & 0 & y_C & 0 & 0 & y_C r_{33} - r_{23} \end{pmatrix}$$

$\delta = [t_1 \ t_2 \ t_3 \ a \ b \ c]^T$, and $X\delta = 0$, the eigenvector associated with the smallest eigenvalue of $X^T X$ is δ . Then we linearly obtain a , b , c and t (up to scale).

IV. EXPERIMENTS

A. Computer Simulation

The simulated camera has a focal length $f = 1600$, unit aspect ratio, zero skew and principal point close to the center of the image. The image resolution is 1200×800 . Three rotation angles in matrix R are random in $[20^\circ, 70^\circ]$, and the simulated cuboid has length ratio 1:2:3. We evaluate the performance with respect to the chosen based points. Add Gaussian noise to the projected image points in pixels, the error is 1.12% for noise of 1.5 pixels, and will increase till 4.39% when noise is 4.5 pixels, which is shown in Figure 2.

B. Real Data

The proposed method was also tested on many real images, some are shown below. We compared the computed results with ground truth measurements. The relative error in all experiments did not exceed 5%. Criminisi et al [4] just measure area and length ratios on parallel planes, our method can directly obtain the measurements between orthogonal/parallel directions. In Figure 3, the edges of windows help us compute the vanishing points. Choose O , A and B , the length ratios between two orthogonal directions can be computed. The reference length is $OA = 65\text{cm}$, the real length on the two directions are directly obtained. Define a projective coordinate system, the metric measurements on the plane can be completely computed. Shown in Figure 4, the measurements between orthogonal planes are computed using three orthogonal vanishing points. Orthogonal planes

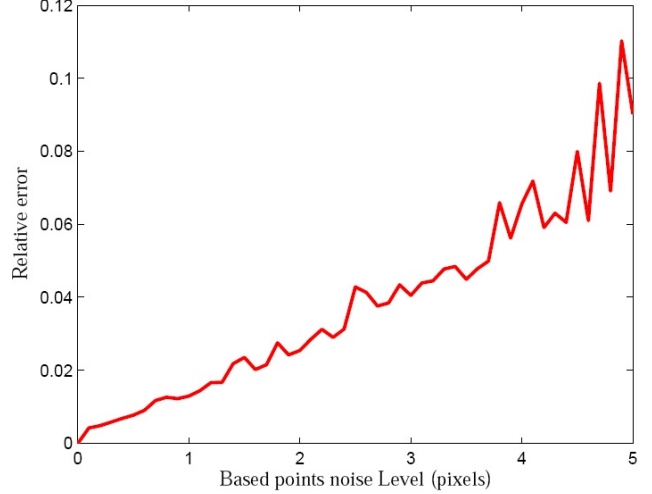


Figure 2. Performance with noise for chosen based points on image.

are in a same homography defined by O, A, B and C , area and length can be compared easily.

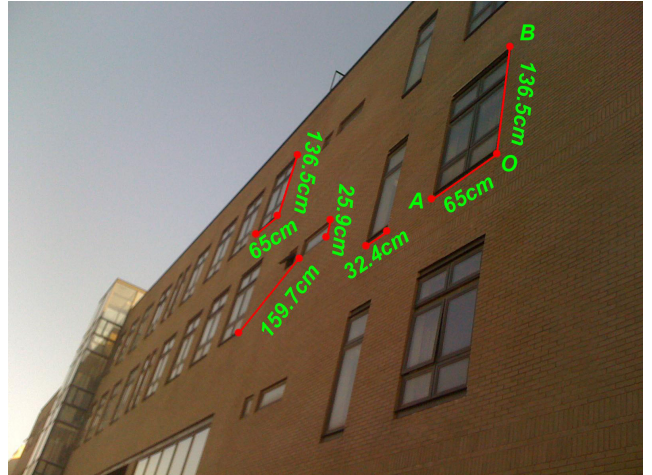


Figure 3. O, A, B are chosen to recover metric measurements. Reference length is $OA = 65\text{cm}$. The metric measurements on the plane can be completely computed, while previous methods just can compute the measurements along vanishing directions.

V. CONCLUSION

We have presented a novel method for single view metrology that just requires orthogonal vanishing points. Previous methods for metrology only compute the length ratios of objects along the same direction, our work is to obtain the length ratios of line segments, which lie on different orthogonal orthogonal directions, then we can recover metric measurements directly from uncalibrated images. The experiments show that our method obtain accurate results. To make algorithm further robust, we can add feature points

or other extra information. In the future, we will investigate how full geometry can be represented and computed from images.

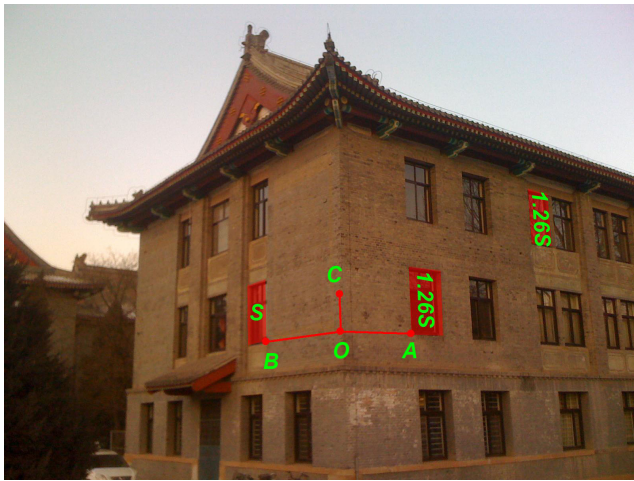


Figure 4. Orthogonal planes are in a same homography defined by O , A , B and C , area and length can be compared easily.

VI. ACKNOWLEDGMENT

This work was supported in part by the NKBRPC 973 Grant No. 2006CB303100, the NHTRDP 863 Grant No. 2009AA01Z329, and the NHTRDP 863 Grant No. 2009AA012105.

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