Compressive Sampling Recovery for Natural Images

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***Abstract*—Compressive sampling (CS) is a novel data collection and coding theory which allows us to recover sparse or compressible signals from a small set of measurements. This paper presents a new model for natural image recovery, in which the smooth l_0 norm and the approximate total-variation (TV) norm are adopted simultaneously. By using one-order gradient decrease, the speed of algorithm for this new model can be guaranteed. Experimental results demonstrate that the principle of the model is correct and the performance is as good as that based on TV model. The computing speed of the proposed method is two orders of magnitude faster than that of interior point method and two times faster than that of the Nesta optimization based on TV model.**

Keywords-compressive sampling;image recovery;TV norm; smooth l_0 norm

I. INTRODUCTION

The recently developed Compressive Sampling (CS) framework [1,2] is a statistical technique of data acquisition and estimation, which allows us to recover sparse or compressible signals from a small set of measurement. The signal's sparsity plays the similar role in CS as that bandwidth in classical Nyquist-Shannon sampling theorem. Its obvious advantage is that it can encode the signal or image fast without any prior knowledge about signal.

According to Nyquist-Shannon sampling theorem usually a huge amount of data are acquired at first and then compressed to cut the storage and transmission burden. CS provides an entirely different way which combines the sampling and compressing at the same time. This feature is especially useful in field of image and video. CS has been used in MRI imaging [3] and single-pixel imaging [4], which provides a potential solution for expensive unseen-light imaging.

In this paper, we focus on the image recovery method which is the crux in CS imaging. The problem can be brought into the convex optimization problem. Many researchers [5-8] have made a lot of outstanding work on this topic by using Basis Pursuit (BP).They have demonstrated the accurate results on the scale of interest (hundreds of thousands of measurements and millions of pixels). Among their work, the total variation (TV) minimization method is the mostly used in image recovery, which belongs to the BP. However solving the BP problem is about 30-50 times expensive than solving the least squares problem hence image blocking [9] was proposed to decrease the scale of solution. It reduces the computational complexity but it is hard to select a suitable sampling rate. Mohimani et al [11] proposed a recovery method based on the smooth l_0 norm which is two or three orders of magnitude faster than BP, but it was only applied to 1-D sparse signal, not applied to images.

In this paper a fast recovery method for natural images is proposed, in which the smooth l_0 norm and the approximate total-variation (TV) norm are adopted simultaneously. It maintains the speed of the smooth l_0, and uses an approximate TV norm to improve the accuracy.

II. IMAGE RECOVERY FROM COMPRESSIVE SAMPLES

The main idea of the CS is to o recover original signal $\mathbf{x} \in \mathbf{R}^N$ from compressive samples $\mathbf{y} \in \mathbf{R}^M$, where $M >> N$, $\mathbf{R}$ is real number set. Suppose that $\mathbf{x} = \mathbf{\Psi\theta}$, where $\mathbf{\theta}$ is sparse or compressible and $S = \|\mathbf{\theta}\|_0$, i.e. the number of nonzero components of $\mathbf{\theta}$.

The measurement can be denoted by

$$\mathbf{y} = \mathbf{Ax} = \mathbf{A\Psi\theta}, \tag{1}$$

where $\mathbf{A}$ is the measurement matrix.

If $\mathbf{A\Psi}$ complies the restricted isometry property (RIP) or uniform uncertainty principle (UUP), the signal $\mathbf{x}$ can be recovered losslessly. Normally we need $3S$ to $5S$ measurements to recover the signal in practice.

The recovery $\mathbf{x}$ from $\mathbf{y}$ can be given by

$$\min \|\mathbf{\theta}\|_0, s.t. \quad \mathbf{y} = \mathbf{A\Psi\theta}. \tag{2}$$

If $\mathbf{\theta}$ is obtained, then signal $\mathbf{x}$ can be recovered.

Since the exact solution to (2) is NP-hard. Candès et al [1] and Donoho[2] proposed a similar program in l_1 norm

$$\min \|\mathbf{\theta}\|_1, s.t. \quad \mathbf{y} = \mathbf{A\Psi\theta}. \tag{3}$$

The l_1 minimization (BP) will cause the ringing artifact around image edges it is not suitable for image recovery. While the TV minimization (min-TV) is the common model in solving images inverse problems, so TV norm is used to

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replace the l_1 norm, and the images recovery problem from compressive samples can be described as

$$\min TV(\mathbf{x}) \quad s.t. \, \mathbf{y}=\mathbf{A}\mathbf{x}$$
$$TV(\mathbf{x})=\sum_{i,j}\sqrt{(x[i+1,j]-x[i,j])^2+(x[i,j+1]-x[i,j])^2} \cdot \quad (4)$$

The problem can be solved as a second-order cone program, which is a standard optimization technology. l_1-magic [12] is a typical algorithm to solve the problem by using interior point methods. The problem can also be solved by Nesta [10], a one-order gradient decrease method, which converges rapidly and faster than interior point optimization while achieving as good performance as the l_1-magic. Although interior point optimization and Nesta can solve the problem, their computational speed is not satisfactory, and the more efficient algorithms are still required.

III. OUR METHOD

We propose a new model for natural images recovery in which the smooth l_0 and the approximate total-variation (TV) norm are applied simultaneously. The modified program is given by

$$\min \; \{\lambda_0 F_0(\mathbf{x}) + \lambda_{TV} F_{TV}(\mathbf{x})\} \quad s.t. \, \mathbf{y} = \mathbf{A}\mathbf{x} \,, \quad (5)$$

where $F_0()$ is the smooth l_0 norm and represents the common requirement for sparse signal. $F_{TV}()$ is approximate TV norm designed to represent the characteristics of natural images. λ_0 and λ_{TV} are the weights which control the contribution of $F_0()$ and $F_{TV}()$. $F_0()$ is originated from Mohimani et al[11] where the discontinuation of l_0 norm results (2) to be NP-hard, so a smooth function is designed to approximate the l_0 norm, such as

$$f_\sigma(x)_0 = \exp(-x^2/2\sigma^2)$$
$$\lim_{\sigma\to 0} f_\sigma(x)_0 = \begin{cases} 1 & if \quad x=0 \,, \\ 0 & else \end{cases} \quad (6)$$

$$\|\mathbf{x}\|_0 \approx N - \sum_i f_\sigma(x_i)_0 \,. \quad (7)$$

For the signal sparse in time domain, by using the smooth l_0 norm, (2) can be rewritten as

$$\max \sum_i f_\sigma(x_i)_0, \quad s.t. \quad \mathbf{A}\mathbf{x} = \mathbf{y} \,. \quad (8)$$

The fast algorithm Sl_0 [11] applying the idea is two orders of magnitude faster than l_1 minimization. Through analyzing the algorithm we find that the algorithm is actually a simulated annealing process, the parameter of σ can be explained as the annealing temperature.

We introduce this idea into natural image recovery, and define the smooth l_0 norm as

$$F_0(\mathbf{x}) = -\sum_i f_\sigma((\mathbf{W}\mathbf{x})_i)_0 \,, \quad (9)$$

where W is a sparse transform, for example, the wavelet transform. Then the natural image can be recovered by solving the following optimization problem:

$$\min F_0(\mathbf{x}) \quad s.t. \, \mathbf{A}\mathbf{x} = \mathbf{y} \,. \quad (10)$$

Unfortunately we find that the quality of recovered image is poor based on (10) only. The reason is that the smooth l_0 has the inherent disadvantage of the l_0 norm where only the non-zero numbers of entries are considered, but their amplitudes are not considered. Since the natural images are compressible instead of sparse, this ignorance to the amplitudes will cause the degradation of the recovered images. To deal with this issue, we use the TV norm in our method as shown in (5).

The TV norm describes the variation of image in the space domain, which can be decomposed into horizontal and vertical dimensions. In order to reduce the complexity of computation, the l_1 norm is used in the definition,

$$\|\mathbf{x}\|_{TV\_HV} = \|\mathbf{H}\mathbf{x}\|_1 + \|\mathbf{V}\mathbf{x}\|_1$$
$$\|\mathbf{x}\|_{TV\_HV} = \sum_{i,j}|x[i+1,j]-x[i,j]| + \sum_{i,j}|x[i,j+1]-x[i,j]| \,, \quad (11)$$

where H,V are horizontal gradient and vertical gradient operator respectively.

In order to leverage the merit of the solution to the smooth l_0, we design an approximate l_1 norm related to temperature σ, which is called the approximate l_1 norm as

$$f_\sigma(x)_1 = |x|(1-\exp(-x^2/2\sigma^2)) \,. \quad (12)$$

So the TV norm can be written as

$$F_{TV}(\mathbf{x}) = \sum_i f_\sigma((\mathbf{H}\mathbf{x})_i)_1 + \sum_i f_\sigma((\mathbf{V}\mathbf{x})_i)_1 \,, \quad (13)$$

which is called the approximate TV norm.

The final algorithm based on (5) is similar to Sl_0 [11]. In the annealing process, the initial estimation of **x** is the least square solution, and the initial temperature σ_0 is two times of the maximum of initial estimation **x**. The parameter σ decreases half each time, when $\sigma > 10$, λ_0 is 1 and λ_{TV} is zero. When $\sigma \le 10$, λ_{TV} is 0.2 and λ_0 is 0.6. The parameters are selected through simulation results. Since only one-order gradient is required to calculate in the whole process, the speed of algorithm is fast.

Figure 1. The original natural images

Figure 2. The recovery images using our method when K=25000

IV. EXPERIMENTAL RESULTS

The performance of the proposed method has been evaluated and compared with the min-TV recovery algorithms. The results of the min-TV are obtained by using l_1-magic package [12] and NESTA package [10]. All experiments have been carried out in Matlab 7.0.1 on Intel T2300 1.66GHz CPU.

The original natural images Lena, Cameraman, Peppers and Boats shown in Figure 1 are used to test various methods. They are sampled by a random Fourier matrix. The real and imaginary parts of coefficients are considered as two measurements. Figure 2 shows the recovery images using our method with K=25000 measurements. Figure 3 shows the average PSNR of recovery images for the five different methods, i.e. reference [8], l_1-magic, Nesta, Sl_0 and our method. The results of Candès and Romberg [8] are set as benchmarks, where a random Fourier sampling matrix was applied directly in the wavelet domain.

Compared with Sl_0, the PSNR of recovery image using our method increases at least 2dB and 5dB if K is bigger than 15000, and the PSNR is as good as or better than those based on TV minimization if K is bigger than 15000. Table 1 tabulates the average running time. Our method has less iteration number and lower computational cost, l_1-magic completes the recovery in thousands seconds, Nesta in 30-50 seconds，but our method is in about 10 seconds.

TABLE I. AVERAGE RUNNING TIME FOR DIFFERENT K

K	Average running time for image recovery(seconds)		
	l_1-magic	***Nesta***	***our method***
10000	2297.2	57.1	10.3
15000	2721.4	48.3	9.8
20000	2168.2	45.9	11.7
25000	2038.3	34.4	11.4

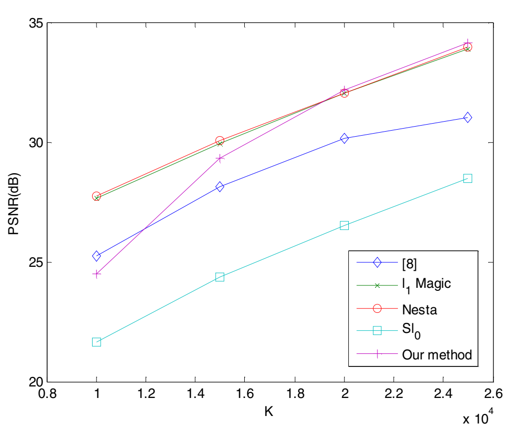

Figure 3. Recovery results for 256×256 images parameters for different K(1D Fourier)

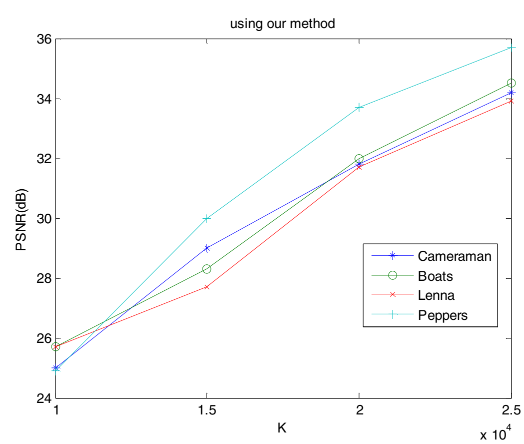

Figure 4. Recovery results for 256×256 images parameters for different K (Noiselet,our method)

Figure 4 gives the PSNR for four 256×256 natural images recovered by our method in the condition that the measurement matrix is a random noiselet sampling matrix. It demonstrates that our method also fits for other measurement matrices.

V. CONCLUSION

This paper has presented a new model for natural image recovery, where we apply the smooth l_0 to replace the l_0 norm, and the approximate TV to represent the variation of image, hence the simulated annealing is adopted to reduce the computational burden. The experimental results show that the proposed method is much faster than l1-magic and Nesta, and the recovery image's quality is as good as these two methods.

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