

LLN-Based Model-Driven Validation of Data Points for Random Sample Consensus Methods

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Abstract

This paper presents an on-the-fly model-driven validation of data points for random sample consensus methods (RANSAC). The novelty resides in the idea that an analysis of the outcomes of previous random model samplings can benefit subsequent samplings. Given a sequence of successful model samplings, information from the inlier sets and the model errors is used to provide a validness of a data point. This validness is used to guide subsequent model samplings, so that the data point with a higher validness has more chance to be selected. To evaluate the performance, the proposed method is applied to the problem of the line model fitting and the estimation of the fundamental matrix. Experimental results confirm that the proposed algorithm improves the performance of RANSAC in terms of the estimate accuracy and the number of samplings.

1. Introduction

Finding reliable data points for model fitting is difficult and critical for many computer vision problems such as stereo matching and motion estimation. Due to phenomena like occlusion, depth discontinuities and repetitive patterns, incorrect matches (outliers) survive after the matching process when only local image descriptors are compared. Therefore, a robust method is essential to detect and remove the mismatches. Robust methods search for the set of matches consistent with some global constraint. To this end, random sample consensus (RANSAC) [1] and its variants have become the methods of choice for outlier removal [2].

The random sample consensus algorithm is a hypothesis and verification algorithm. It repeatedly generates estimates (hypothesis) from minimal sets of data points, randomly selected from the entire set

(model sampling), and then tests each estimate for support from the entire set (model verification). The standard RANSAC [1] follows uniform model sampling strategy that each datum has the equal chance to be selected. The minimal sets are selected uniformly over all data. As the dimensionality of the minimal set increases, the efficacy of uniform model sampling degrades exponentially [3]. Since each datum has different reliability, it is expected that a reliable datum be more likely selected using non-uniform model sampling.

To improve the random model sampling of RANSAC, different methods have been proposed to take advantage of information other than original data. Tordoff and Murray proposed to use matching priors from the matching process to guide the model sampling [4]. A similar idea was followed by PROSAC where the model sampling is based on a subset of “good” data, defined by the similarity from the matching process [5]. Myatt et al. proposed a biased model sampling, where only one data point was selected based on the uniform distribution, and consecutive data points were found within a hypersphere of radius relative to the Euclidean distance to the initial data point [6]. Pylvaininen and Fan proposed a weighting method, hill-climbing (HC) algorithm [7], in which the probability of selection in a next model sampling was increased proportional to the number of inliers associated with the current favored model. The HC algorithm may get stuck climbing a local maximum since it may overly emphasize some data over others. To address this shortcoming, the same authors suggested an EIS-Metropolis algorithm, wherein the data weights guiding the data selection are derived from statistics of repeated inlier points, i.e., Ensemble Inlier Sets (EIS), accumulated from previously proposed models [8].

Recently, motivated from the Law of Large Number (LLN), we independently developed two methods for

the on-the-fly validation of data points [9][10]. Different from the EIS-Metropolis algorithm, we exploited the delayed weight onset strategy for model samplings to reduce the effect of successful but unreliable model samplings and counted inliers only from the favored models with the best-so-far scores. Our contribution here follows our previous works. The method presented in this paper is similar in spirit, but in contrast to [9][10], the validity of an individual data point is derived from the errors related to the favored models with the best-so-far scores. This modification improves the capability of dealing with local optima for the high dimensional fitting problem with various scoring techniques.

This paper is organized as follows. Section 2 presents the proposed validation method. Section 3 describes the entire robust algorithm, called LLN-MD RANSAC. Experimental results, obtained from the line model fitting and the estimation of the fundamental matrix, are shown in Section 4. The last section concludes the paper.

2. Validation of data points

2.1. Terms and definitions

We define a model as a parameterized function $f(\mathbf{x}|\theta)$, where \mathbf{x} is a data point and θ are the model parameters. A data point is either an image-point in the case of line model fitting, or a pair of corresponding points in the case of two-view geometry. All possible data points form a data set \mathcal{S} . A point $\mathbf{x} \in \mathcal{S}$ is said to perfectly fit the model when $f(\mathbf{x}|\theta)=0$. Let k be an index of one model sampling process that randomly selects a minimal subset of \mathcal{S} with enough points to define a model θ . For instance, in the line model fitting, two image-points in \mathcal{S} would build a minimal subset, while seven correspondent pairs would constitute a minimal subset in the estimation of the fundamental matrix.

Given a parameter ε , an inlier set for the model θ_k at the k -th model sampling is

$$I_\varepsilon(\theta_k) = \{ \mathbf{x} \in \mathcal{S} | f(\mathbf{x}|\theta_k) < \varepsilon \}. \quad (1)$$

$N(I_\varepsilon(\theta_k))$ is the number of inliers in $I_\varepsilon(\theta_k)$. Given a model θ , we also define the validity $P_v(\mathbf{x}|\theta)$ of a data point \mathbf{x} as the probability of it being an inlier

$$P_v(\mathbf{x}|\theta) = P(\mathbf{x} \in \mathcal{S} | f(\mathbf{x}|\theta) < \varepsilon) \quad (2)$$

and a model error

$$e(\mathbf{x}|\theta) = f(\mathbf{x}|\theta). \quad (3)$$

2.2. LLN-based validation

An individual model sampling process is a random and independent event. Probability theory indicates that if an event is repeated many times the sequences of the random events will exhibit certain statistical patterns, which can be studied and predicted. This

suggests that the validity of a data point $P_v(\mathbf{x}|\theta)$ could be estimated from the outcomes of the previously completed K model samplings. Based on the law of large number, the validity of a data point $P_v(\mathbf{x}|\theta)$ can be determined from

$$P_v(\mathbf{x}|\theta) = O\{ P_v(\mathbf{x}|\theta_k), k \in [1, K] \}, \quad (4)$$

where $P_v(\mathbf{x}|\theta_k)$ is the validity of a data point \mathbf{x} , given the model estimate θ_k at the k -th model sampling process and O is an operation to derive a final estimate from previous K outcomes.

The validity of a data point is defined as the probability of it being an inlier, which fits the model θ to be estimated. From the entire process of RANSAC, only the model estimate θ_k with the score $C(\theta_k)$, which is not less than anyone before (successful model sampling), is accepted as the estimate of the ground-truth model θ at the k -th model sampling. Otherwise, the model estimate θ_k is discarded. Let m be an index of successful model samplings and Θ be the subset of the model estimates obtained from M successful model samplings

$$\Theta = \{ \theta_1, \theta_2, \dots, \theta_m, \dots, \theta_M \} \quad (5)$$

with the scores

$$C(\theta_i) \leq C(\theta_j) \quad \text{if } i < j \quad (6)$$

Then, eq. (4) is modified as

$$P_v(\mathbf{x}|\theta) = O\{ P_v(\mathbf{x}|\theta_k), \theta_k \in \Theta \}. \quad (7)$$

The operation O in equation (7) is a rule to derive a final estimate of $P_v(\mathbf{x}|\theta)$ from the outcomes of M previous successful model samplings Θ . Since each model sampling process is a random and independent event, one simple operation O is to take an average and equation (7) can then be written as

$$P_v(\mathbf{x}|\theta) = \frac{1}{M} \sum_{m=1}^M P_v(\mathbf{x}|\theta_m) \quad (8)$$

2.3. Calculation of the model-driven validity

How to calculate the value of $P_v(\mathbf{x}|\theta_m)$ in eq. (8) is the last step for the validation of a data point \mathbf{x} . Recall that the inlier set $I_\varepsilon(\theta_m)$ and the model error $e(\mathbf{x}|\theta_m)$ are two outcomes related to the validity of a data point \mathbf{x} after an estimate θ_m of the model θ has been reached. A small model error $e(\mathbf{x}|\theta_m)$ indicates that the data point \mathbf{x} fits the model estimate better than data points with a large model error.

Different from our previous work [10] that binarized the value of $P_v(\mathbf{x}|\theta_m)$ according to the inlier set $I_\varepsilon(\theta_m)$, We determine $P_v(\mathbf{x}|\theta_m)$ from the model error $e(\mathbf{x}|\theta_m)$ by the evaluation of how well a data point fits the model estimate θ_m and name this method a model-driven validation, i.e.,

$$P_v(\mathbf{x}|\theta_m) \leftarrow P_v(\mathbf{x}|e(\mathbf{x}|\theta_m)). \quad (9)$$

One possible calculation of $P_v(\mathbf{x}|e(\mathbf{x}|\theta_m))$ was proposed in our previous work [9], i.e.,

$$P_v(\mathbf{x}|e(\mathbf{x}|\theta_m)) = 1 - P(0 \leq e < e(\mathbf{x}|\theta_m)). \quad (10)$$

where $P(0 \leq e < e(\mathbf{x}|\theta_m))$ is the probability of the observed error e lying in the interval $[0, e(\mathbf{x}|\theta_m))$. Similar to the calculation proposed in [9], we quantize possible values of e into a finite number J of intervals $[Q_l, Q_{l+1})$ with each interval having the same size. Let $Q_{l,c}$ be the midpoint of the interval (Q_l, Q_{l+1}) . Then, the probability distribution $p'(Q_l \leq e < Q_{l+1})$ can be determined from [9]

$$p'(Q_l \leq e < Q_{l+1}) \propto H(Q_l, Q_{l+1}) \times e^{-Q_{l,c}^2 / (2\sigma_r^2)} \quad (11)$$

where $H(Q_l, Q_{l+1})$ is the number of errors $e(\mathbf{x}|\theta_m)$ in the interval (Q_l, Q_{l+1}) . Different from that in [9], we further take into account the quality, measured by the inlier number $N(I_\varepsilon(\theta_m))$, of the model estimate θ_m , i.e.,

$$p'(Q_l \leq e < Q_{l+1}) \propto \begin{cases} N(I_\varepsilon(\theta_m))H(Q_l, Q_{l+1})e^{-Q_{l,c}^2 / (2\sigma_r^2)} & \text{if } Q_{l+1} < \varepsilon \\ H(Q_l, Q_{l+1})e^{-Q_{l,c}^2 / (2\sigma_r^2)} & \text{if } Q_{l+1} \geq \varepsilon \end{cases} \quad (12)$$

Normalizing $p'(Q_l \leq e < Q_{l+1})$ results in

$$p(Q_l \leq e < Q_{l+1}) = \frac{p'(Q_l \leq e < Q_{l+1})}{\sum_{l=0}^L p'(Q_l \leq e < Q_{l+1})}. \quad (13)$$

Consequently, the probability $P(0 \leq e < e(\mathbf{x}|\theta_m))$ is

$$P(0 \leq e < e(\mathbf{x}|\theta_m)) = \sum_{l=0}^L p(Q_l \leq e < Q_{l+1}) \quad (14)$$

with L determined by comparing the error $e(\mathbf{x}|\theta_m)$ with the intervals so that $Q_L \leq e(\mathbf{x}|\theta_m) < Q_{L+1}$.

3. Entire LLN-MD RANSAC

The proposed validation algorithm can be combined with any RANSAC-like robust methods. Algorithm 1 described below is an example of the proposed validation of data points together with the standard RANSAC to build the so-called LLN-MD RANSAC method. Let I_{max} be the maximum number of model samplings to be performed. At the beginning of the process, all values of $P_v(\mathbf{x}|\theta)$ for each data point \mathbf{x} are assumed to be equal and chosen to be 0.5. For each model sampling, a minimal subset of s data points is randomly selected by the Monte-Carlo method according to $P_v(\mathbf{x}|\theta)$ [2]. These data points provide a model estimate θ_k . After that, we verify the model estimate θ_k and determine the inlier set $I_\varepsilon(\theta_k)$. If $N(I_\varepsilon(\theta_k)) > N(I_\varepsilon(\theta_m))$, where m is a previous successful model sampling, we save this model estimate and set $\theta_{m+1} = \theta_k$. Each data point will be validated based on all favored model estimates so far. However, the values of $P_v(\mathbf{x}|\theta)$ will be updated only if the current series k of model sampling is larger than a pre-defined number I_r to reduce negative effect of successful but unreliable model samplings at the beginning of the process. In the experiment, I_r is chosen to be $0.15 \times I_{max}$. This process

will continue until the upper limit on the number of samples is reached. The consensus with the best score is kept for deriving the final estimate of the model θ .

Algorithm 1: LLN-MD RANSAC

1. Initialize $P_v(\mathbf{x}|\theta)$ of each data point to 0.5. Set $N(I_\varepsilon(\theta_0))=0, k=1$, and $m=0$;
2. Extract s data points by the Monte-Carlo method according to $P_v(\mathbf{x}|\theta)$ and compute the model θ_k using these data points;
3. if $N(I_\varepsilon(\theta_k)) > N(I_\varepsilon(\theta_m))$ set $\theta_{m+1} = \theta_k$; else go to step 6;
4. set $I_r = 0.15 \times I_{max}$;
5. if $k \geq I_r$, update $P_v(\mathbf{x}|\theta)$ using eq. (8);
6. increase k by 1 and repeat steps 2-5 until maximum number of samplings I_{max} has been reached;
7. re-compute the model θ using the inlier set $I_\varepsilon(\theta_M)$.

4. Experimental Results

The proposed LLN-MD RANSAC method was compared to both the standard RANSAC [1] and the RANSAC-EIS-Metropolis [8]. For each given outlier ratio, 200 tests were performed in order to achieve statistically meaningful comparisons. The estimate accuracy was determined by how well the ground-truth data fit the optimal model estimate, obtained from all inliers, as measured by both the algebraic error in the line model fitting and the Sampson error in the estimation of the fundamental matrix (F matrix).

4.1. Results of line model fitting

Each test randomly generated a line model. Along the line segment, 200 ground truth image-points were randomly selected. Gaussian noise with a distribution $N(0, I)$ was added to these 200 ground truth points. A randomly selected subset of points, to be considered as outliers in the test, was further corrupted by a uniform noise so that the outlier had a point-to-line distance between 10 and 200 pixels. Tables I and II show the experimental results averaged over such 200 tests for each ratio of outliers ranging from 5% to 45%.

The results of the line model fitting confirm that the proposed LLN-MD RANSAC performs better than either the standard RANSAC or the RANSAC-EIS-Metropolis with regards to the estimate accuracy and the total number of model sampling, which is required to achieve that accuracy. The proposed method reduced on average the estimate error by around 7% and the total number of model sampling by around 8% when compared to the standard RANSAC. However, the RANSAC-EIS-Metropolis seems unsuccessful. The reason may be that the RANSAC-EIS-metropolis method accumulates the inliers with the less-than-best scores for weighting data points. Based on the spirit of the RANSAC algorithm, the inliers with the less-than-

best scores do not correspond to the model to be expected and should be discarded. Therefore, counting those inliers with the less-than-best score will reduce the correctness of the validity measurement.

Table I. Estimate accuracy (Algebraic error)

Outlier Ratio	RANSAC	EIS-Metropolis	LLN-MD
5%	0.364	2.091	0.341
15%	0.364	8.641	0.365
25%	0.368	19.57	0.349
35%	0.383	15.55	0.346
45%	0.385	24.12	0.340
Average	0.3728	13.994	0.3482

Table II. Averaged total number of model sampling

Outlier Ratio	RANSAC	EIS-Metropolis	LLN-MD
5%	4.26	267.6	4.02
15%	5.90	1574	5.63
25%	8.51	3738	7.71
35%	12.1	3163	11.0
45%	17.3	5724	15.8
Average	9.61	2893	8.83

4.2. Results of the F-matrix estimation

We further tested the algorithms' performance with a real image sequence *Corridor* [11] for the estimation of the F matrix. Please refer to [10] regarding the generation of inliers and outliers, given an outlier ratio, for each test. Tables III and IV show the experimental results averaged over 200 tests and further over 10 image pairs for each ratio of outliers ranging from 5% to 45%.

Table III. Estimate accuracy (Sampson error)

Outlier ratio	RANSAC	EIS-Metropolis	LLN-MD
5%	0.534	0.516	0.506
15%	0.594	0.561	0.519
25%	0.62	0.515	0.511
35%	0.631	0.496	0.494
45%	0.628	0.491	0.489
Average	0.6014	0.5158	0.5038

Table IV. Averaged total number of model sampling

Outlier ratio	RANSAC	EIS-Metropolis	LLN-MD
5%	20.36	20.07	19.06
15%	41.59	38.27	36.39
25%	90.87	75.54	74.61
35%	223.7	172.9	172.1
45%	626.8	473.8	470.2
Average	200.66	156.12	154.47

The results of the estimation of the fundamental matrix confirm that the proposed LLN-MD RANSAC is slightly better than the RANSAC-EIS-Metropolis method. In comparison with the standard RANSAC, the proposed method reduced on average the estimate error by around 16% and the total number of model sampling by around 23%.

5. Conclusions

The paper concludes that information accumulated from the previously completed model samplings is useful in guiding subsequent model samplings as indicated by the Law of Large Number. It can improve the performance of RANSAC-like robust estimators in terms of the estimate accuracy and the total number of samplings.

7. References

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